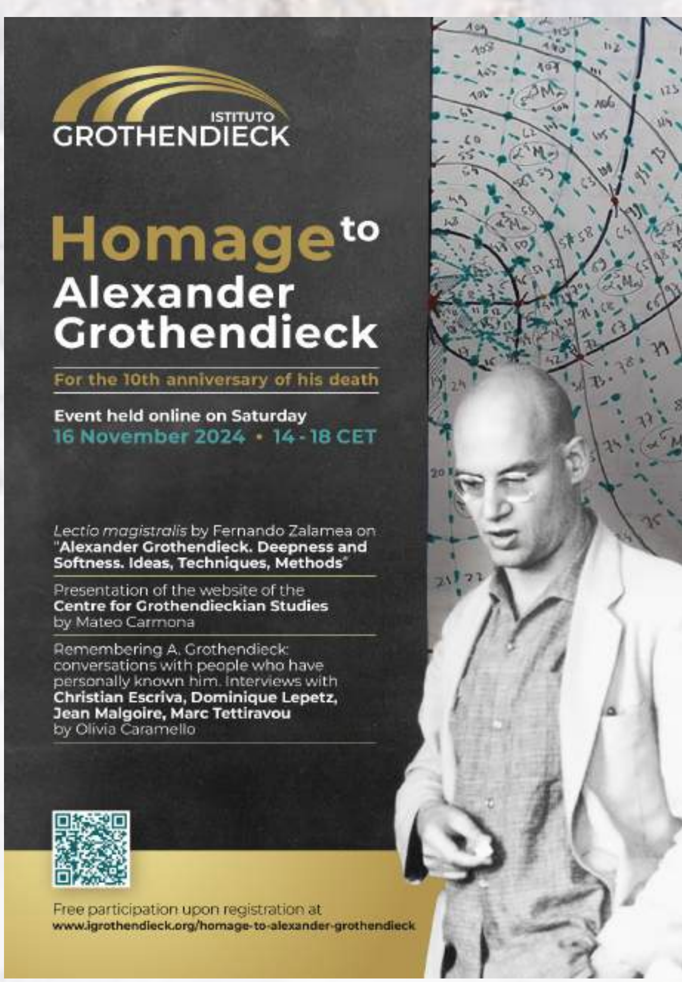
ISTITUTO GROTHENDIECK
CENTRO DI STUDI GROTHENDIECKIANI

On Occasion of the10th Anniversary of Grothendieck's Death (13 November 2024)



ALEXANDER GROTHENDIECK. DEEPNESS AND SOFTNESS

TECHNIQUES – IDEAS – METHODS

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Dedicated to the Memory of Pierre Cartier (1932-2024)

CONTENTS

0.1. Life (a)

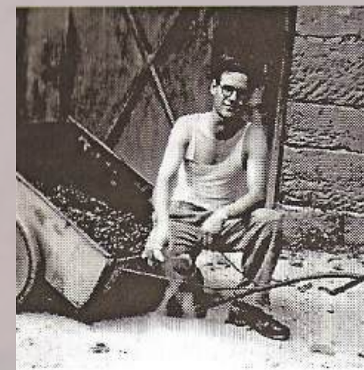
back-and-forth between margins and center

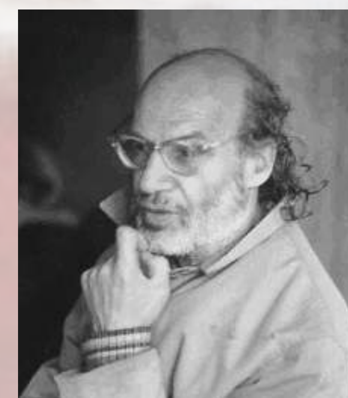
• 1928 – Berlin:	Birth (March 28)
• 1933-39 – Hamburg:	Infancy, care under Lutheran Father Heydorn
• 1940-42 – Rieucros:	Adolescence, concentration camp
• 1942-45 – Chambon:	High school, care under Lutheran Father Trocmé
• 1945-48 – Montpellier:	Career (mathematics, “incredible capacity, unbalanced by suffering”)

• 1948 – Paris:	Initiation to higher mathematics, at Séminaire Cartan
• 1949-53 – Nancy:	Ph. D. (under Dieudonné, Schwartz)
• 1953-54 – Sao Paulo:	Postdoc (topological vector spaces)
• 1955 – Kansas:	Postdoc (homological algebra)
• **1959-70 – Paris**:	**THE GREAT CENTER:** Professor at IHES, created specially for him; Fields Medal 1966

• 1970:	Retires IHES for political reasons; travels to Vietnam; extense activism in ecologist groups
• 1970-83:	Partial positions at the Collège de France and Université de Montpellier
• 1983-87:	*Esquisse d’un programme*; *Récoltes et Semailles*, *La Clef des Songes*
• 1980-90:	Long manuscripts; rejects the Crafoord Prize
• 1991:	Disappears from the community and retires in small Pyrenean villages
• 2014:	Death in Saint-Girons (November 13)

0.1. Life (b)





0.2. Work (Multiplicity)

[1953a] Alexander Grothendieck, *Produits tensoriels topologiques et espaces nucléaires*, American Mathematical Society Memoirs 16, 1955 (Ph. D. Thesis, Nancy, 1953).

[1953b] Alexander Grothendieck, *Topological Vector Spaces*, New York: Gordon and Breach, 1973 (Seminar, Sao Paulo, 1953).

[1953c] Alexander Grothendieck, "Résumé de la théorie métrique des produits tensoriels topologiques", *Bol. Soc. Mat. Sao Paulo* 8 (1956): 1-79 (written in 1953).

[1957a] Alexander Grothendieck, "Sur quelques points d'algèbre homologique", *Tohoku Math. Journal* 9 (1957): 119-221 (writing begun in 1955).

[1957b] Alexander Grothendieck, "Classes de faisceaux et théorème de Riemann-Roch" (written in 1957, published in [1960-1969, vol. 6, pp. 20-71]).

[1958a] Armand Borel & Jean-Pierre Serre, "Le théorème de Riemann-Roch (d'après des résultats inédits de A. Grothendieck)", *Bull. Soc. Math. France* 86 (1958): 97-136.

[1958b] Alexander Grothendieck, "The cohomology theory of abstract algebraic varieties", in: *Proceedings International Congress of Mathematics Edinburgh 1958*, Cambridge: Cambridge University Press, 1960, pp. 103-118.

[1960] Alexander Grothendieck, "Techniques de construction en géométrie analytique I-X", *Séminaire Henri Cartan*, volume 13, Paris: Secrétariat Mathématique, 1960-61.

[1960-67] Alexander Grothendieck (with Jean Dieudonné), *Éléments de Géometrie Algébrique*, IV volumes (8 parts), Paris: IHES, 1960-1967.

[1960-69] Alexander Grothendieck (with diverse authors), *Séminaire de Géométrie Algébrique du Bois-Marie*, VII volumes (12 parts), Berlin: Springer, 1970-1973 (original multicopies, 1960-1969).

[1965-70] Alexander Grothendieck, *Motifs*, manuscript, 24 pp.

[1981] Alexander Grothendieck, *La Longue Marche à travers la Théorie de Galois*, manuscript, 1600 pp.

[1983a] Alexander Grothendieck, *Esquisse d'un programme*, manuscript, 57 pp.

[1983b] Alexander Grothendieck, *Pursuing stacks*, manuscript, 629 pp.

[1985-86] Alexander Grothendieck, *Récoltes et Semailles*, manuscript, 1252 pp.

[1987] Alexander Grothendieck, *La Clef des Songes*, manuscript, 315 pp.

[1990-] More than 50.000 pages of manuscripts to be sorted out (BNF).

0.2. Work (Unity)

GERMINACIÓN
(TERCERAS "COSECHAS")

REFLEXIÓN
La clef des songes (1987)
Récoltes et semailles (1986)

1991
LOS FUNDAMENTOS "ARQUETÍPICOS"
Les dérivateurs
(B - C - F - J - K - L - Y - δ - γ)

1984
LA CONDENSACIÓN Y LA VISIÓN
Esquisse d'un programme
(B - C - G - I - J - K - L - X - Y - α - β - δ - γ)

1983
Pursuing Stacks
(B - C - G - L - Y - Z - δ - γ)

1981
LA GRAN RENOVACIÓN
La longue marche à travers la théorie de Galois
(B - C - G - H - I - J - K - L - X - Y - α - β - δ)

(α) espacios moduli de superficies de Riemann (β) grupo de Galois absoluto (δ) grupoide fundamental (γ) tipos homotopía

1975-76
LOS GÉRMENES FINALES
(TERCERAS "SIEMBRAS")

(X) esquemas (Y) topologías de Grothendieck, topos (Z) topos *étale* de un esquema

1958-62
LOS GÉRMENES MEDIOS
(SEGUNDAS "SIEMBRAS")

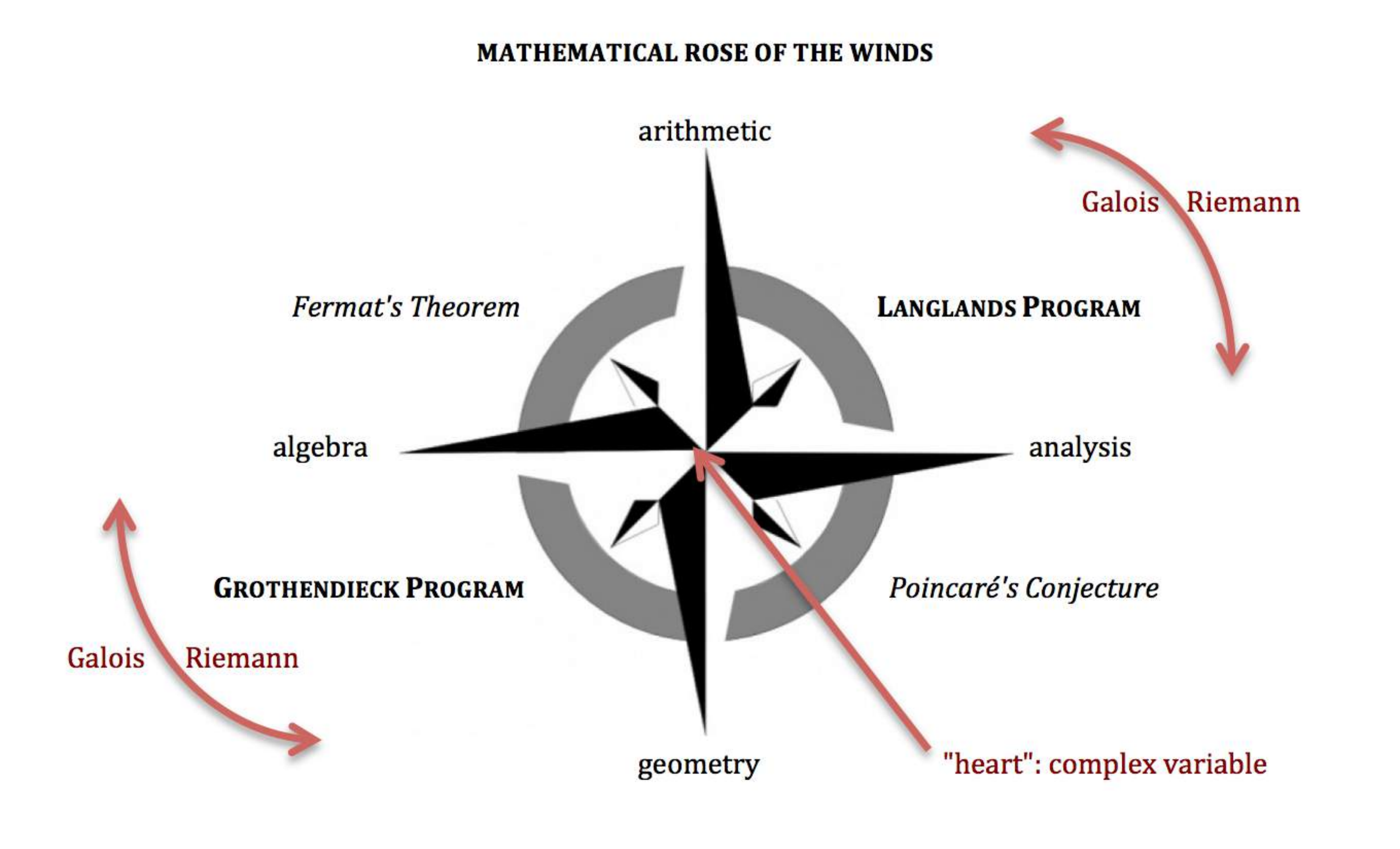
TOPOLOGÍA	ANÁLISIS FUNCIONAL	VARIABLE COMPLEJA	ÁLGEBRA
(A) evtlc	(D) espacios Hilbert/Banach	(G) Riemann-Roch clásico	(J) productos tensoriales
(B) haces	(E) espacios nucleares	(H) espacios de funciones holomorfas	(K) propiedades permanencia
(C) (no) separación	(F) aproximación	(I) esfera de Riemann	(L) co/homología

1950-1951
LOS GÉRMENES INICIALES
(PRIMERAS "SIEMBRAS")

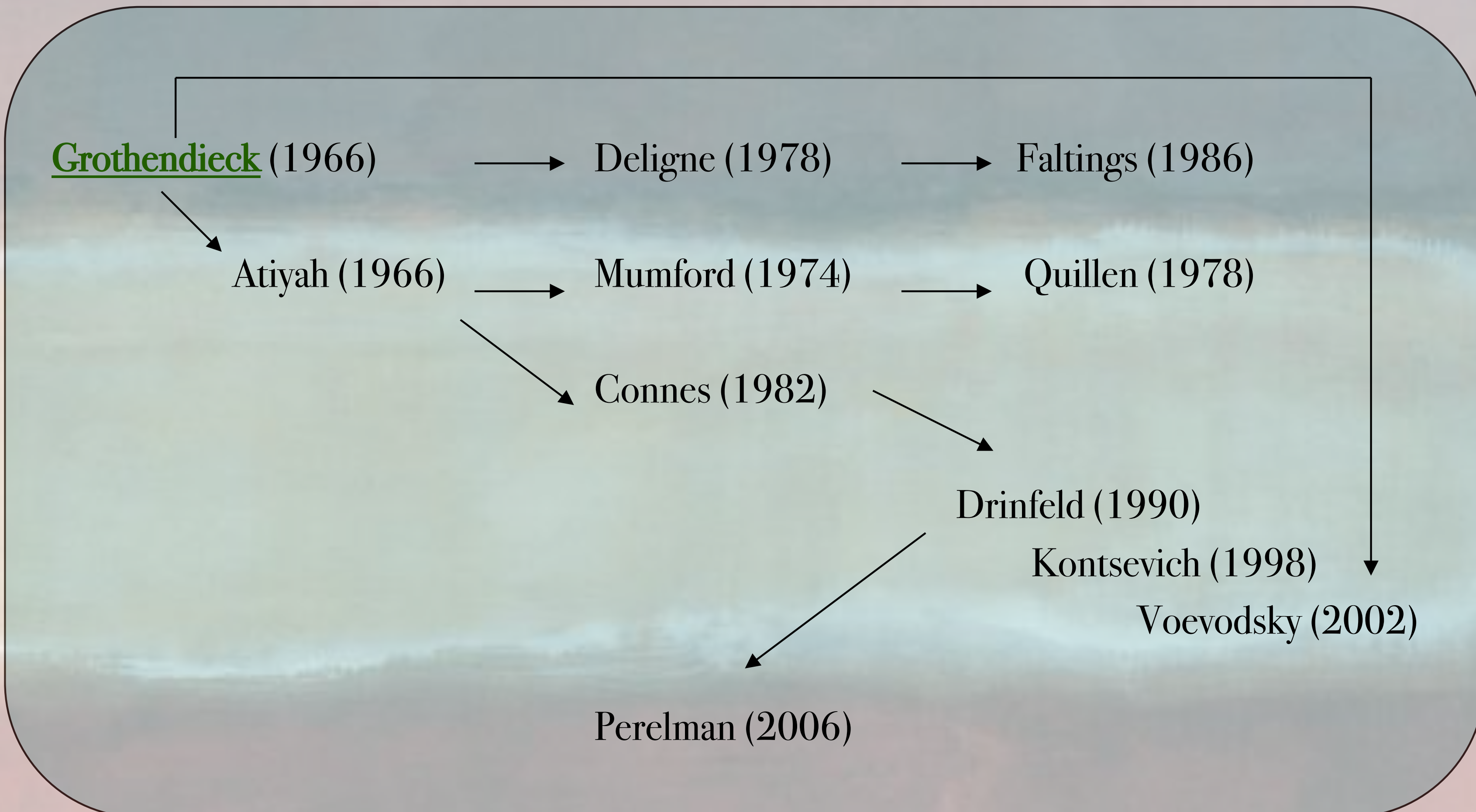
CALIBRACIÓN DE FUERZAS
(# apariciones en los terceros entornos de "germinación")

A (0)	D (0)	G (3)	J (3)	X (2)
B (4)	E (0)	H (1)	K (3)	**Y (4)**
C (4)	F (1)	I (2)	**L (4)**	Z (1)
α(2)	β(2)	**δ(4)**	γ(3)	

0.3. Mathematical influence: the great contemporary programs



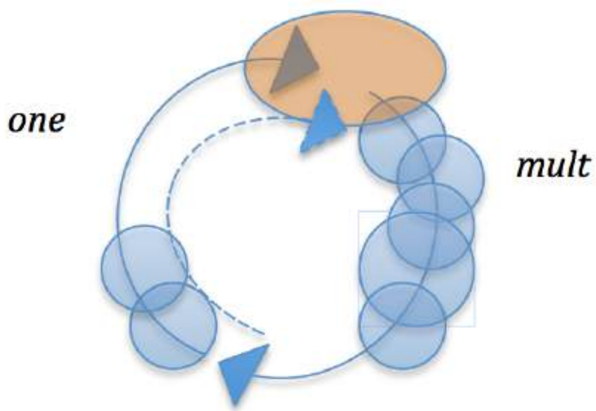
0.4. Mathematical influence: the Fields panorama



1.1. The situation (a): generalization of transits

XIXth century

ANALYTIC CONTINUATION
(Riemann)

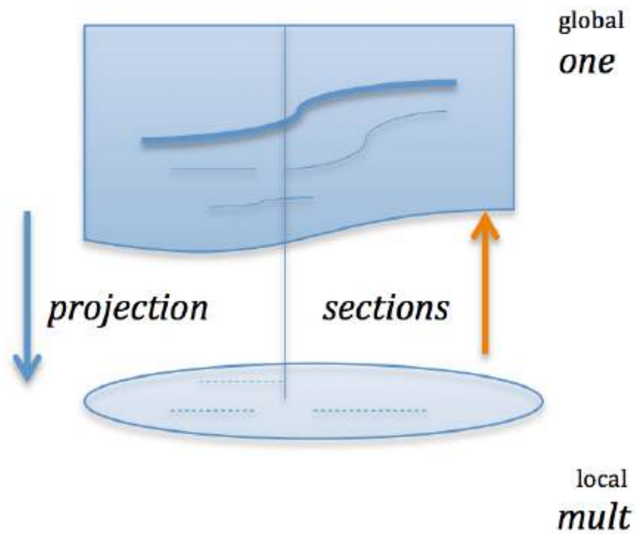


main obstruction:
glued function may be **multivalued**

obstruction solved if
- gluing is independent of **path**
- region is simply connected
(Monodromy Theorem)

XXth century

SHEAVES
(Leray, Cartan, Serre)



main obstruction:
global sections may not exist

space of germs of analytic functions
is Hausdorff
(because of analytic continuation)
general sheaves **NON-SEPARATED**

1.2. The *Tôhoku* (1955)

1955-56	Work: Kansas 1955 - París 1956 (see *Grothendieck-Serre Correspondence*, SMF 2001)
1957	**[1957] "Sur quelques points d'algèbre homologique"** *Tôhoku Math. J.* 9 (1957): 119-221 (Tannaka, editor).

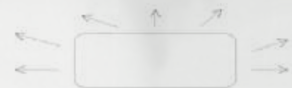
(1) back-and-forth between the Many and the One

NOT: defining *an* object and exploring an *external* structure on the object — Obj + Str — particular

YES: defining the **category** of *all* objects and exploring the *internal* structure of the category — Cat – Str — general

NOT: understanding an object "in-itself" (internal object X)

YES: understanding "in-otherness" (external **representable functor** h_X)



(2) "common frame"

"formal analogy" between
cohomology with coefficients on a sheaf (RIEMANN's heritage)
series of derived functors from functors of modules (GALOIS' heritage)

natural bridges between
algebraic geometry, topology, complex variable, (co)homology

basis: **sheaf theory**
abelian categories (canonical example: categories of abelian sheaves)

(3) infinitary methods (Cantor's "paradise")

axioms **AB3-AB6**: products and arbitrary inductive limits
generator: internal *integration* (in *one* object) of external *differentiation* (in *all* the category)

AB5 + generator: existence of enough injectives
abelianess+ injectivity: construction of derived and cohomological functors

1.3. The *Riemann-Roch Report* (1957)

1957	**[1957] "Classes de faisceaux et théorème de Riemann-Roch" (Rapport Riemann-Roch, RRR)** (November 1 1957), reprinted in : *SGA6*, pp. 20-77.
1958	**[1958] Armand Borel & Jean-Pierre Serre, "Le théorème de Riemann-Roch (d'après des résultats inédits de A. Grothendieck)",** *Bulletin de la Société Mathématique de France* 86 (1958): 97-136.

CLASSICAL RIEMANN-ROCH

Theory of abelian functions [Riemann 1857, Section V]

HARMONIC CONJUGATION

- of POSITIVE (transits) and NEGATIVE (obstructions)
- of SPACE (geometry) and NUMBER (algebra)

the study of ONE function is determined by the study of a MULTIPLICITY of functions *over the Riemann Surface* of the original function

analytic function f
→ **Riemann surface** S associated
→ system of points P_i on S affected with multiplicities m_i, $m=\sum m_i$
→ **vector space** H of holomorphic functions with m assigned *zeros* on S
→ vector space M of meromorphic functions with m assigned *poles* on S

$$m - genus(f) + 1 \quad = \quad dim\,(M) - dim(H)$$

geometric invariant — algebraic harmonics

RIEMANN-ROCH-SERRE

sheaf Θ of (germs of) meromorphic functions with assigned poles
→ cohomology groups $H^0(\Theta)$, $H^1(\Theta)$ (finite-dimensional complex vector spaces)
sheaf Ω of (germs of) meromorphic functions with assigned zeros
→ Serre's **duality theorem**: $H^1(\Theta) \approx (H^0(\Omega))^*$

$$m - genus(f) + 1 \quad = \quad dim\,(H^0(\Theta)) - dim(H^1(\Theta))$$
$$= \quad dim\,(H^0(\Theta)) - dim(H^0(\Omega))$$

geometric invariant — cohomological equilibrium

RIEMANN-ROCH-HIRZEBRUCH

in a fixed variety X with "good" properties,
weaving between Chern classes ("additive" exponential invariants in $H^*(X)$),
Todd classes ("multiplicative" polynomial invariants in $H^*(X)$)
and alternated sums of cohomological dimensions

RIEMANN-ROCH-GROTHENDIECK

- **linearization**: emergence of **K-theory**, with group $K(X)$, ring $A(X)$
- **naturalization**: Chern and Todd classes as transformations between K y H^*
- **relativization**: Serre-Hirzebruch in the case of a morphism $f: X \to Y$ (base variation)

1.4. The diagrammatic imagination (a): types and archetypes

universal constructions and new notions of equivalence

	NOT: (∃) ≈	YES: (∃!) ~
"metaphysical" inversion:	(types)	(archetypes)
methodological inversion:	(statics)	(dynamics)

the archetype (= ***arkhê***; *ark-* Greek root; *arkeô* = move away; *akhô* = found; *arkhên* = project) emerges through the *transformations* of types and the associated *invariants* of transformations ("Serre's C-language", quotient categories, and Grothendieck's **variation over the base**)

RRG: obstruction to commutativity

$$f_*(ch_Y(x)\,\mathfrak{T}(Y)) = ch_X(f_*(x))\,\mathfrak{T}(X)$$

multiplication
naturalization
smoothness

non-commutative adjustement
(**archetype**)

↙ ↘

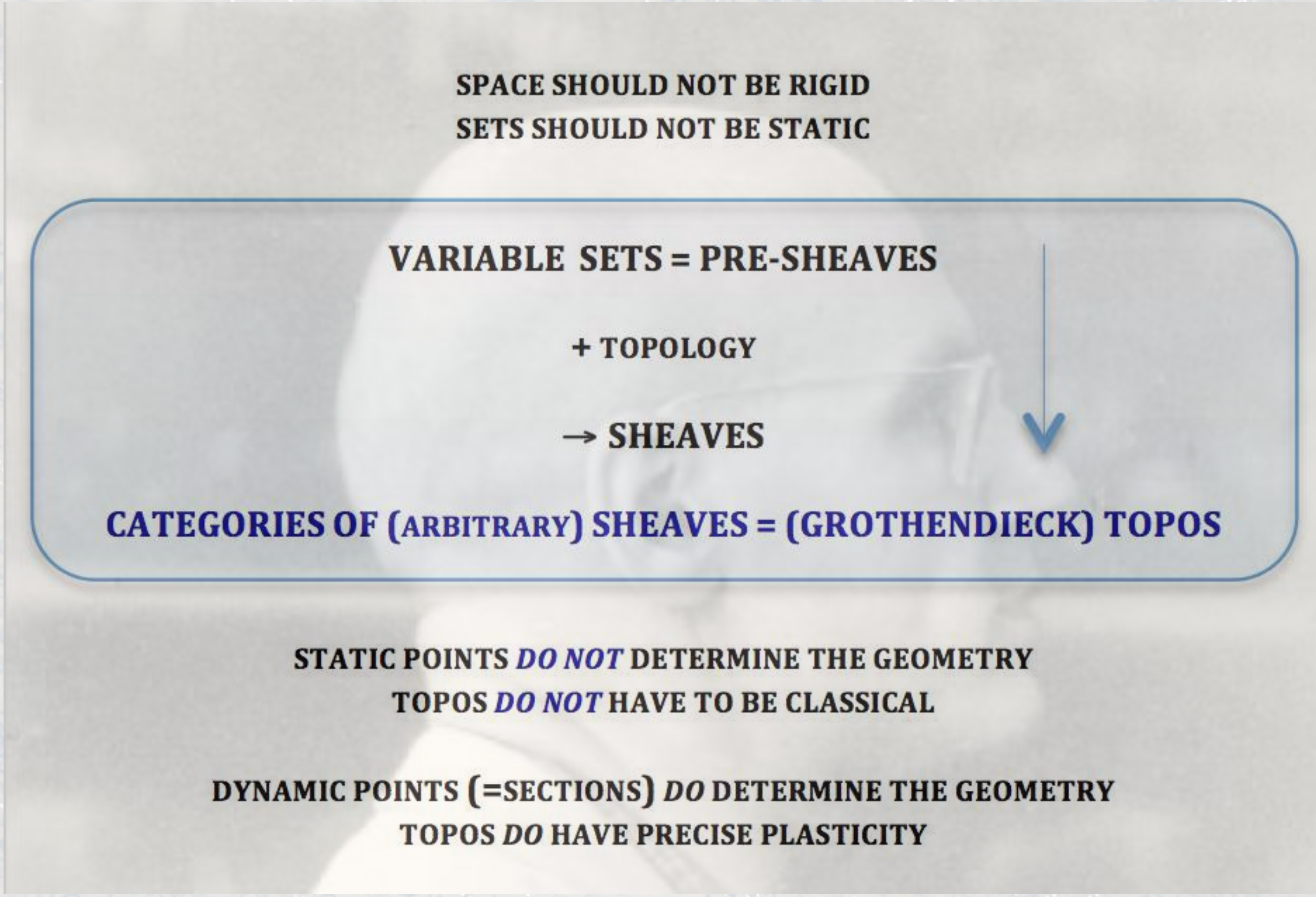
space **TYPES** (geometric genus)
number **TYPES** (algebraic dimension)

2.1. The situation (b): generalization of number

DISCR ≡≡≡≡≡≡≡ CONT	:	Towards a resolution of Weil conjectures		
modern mathematics (1830-1950)	:	algebraic varieties	≡≡≡≡≡≡≡	topologies
contemporary mathematics (1950 –)	:	schemes	≡≡≡≡≡≡≡	topos

Riemann		Galois - Dedekind		Grothendieck	
curve X	$M(X)$ ring meromorphic	variety V / k	*Spec (V)* maximal ideals	comm. unit. ring A	*Spec(A)* prime ideals – Zariski top. sheaves over *Spec(A)*
number (dimension)		*number (extension)*		*number (ramification)*	

2.1. The situation (c): generalization of space



In particular, the **logic of (pre)sheaves is intuitionistic, non classical**

Example: topos of monoid actions: Set^M Boolean iff M group.

2.2. The *Edinburgh ICM Lecture* (1958)

1958 **[1958] "The cohomology theory of abstract algebraic varieties"**, in: *Proceedings International Congress of Mathematics Edinburgh 1958*, Cambridge: Cambridge University Press, 1960, pp. 103-118.

DISCR ---------------- **CONT**

SERRE - CHEVALLEY - NAGATA (1955-56): introduction of "schémas", *gluing* affine spaces

GROTHENDIECK (1958): generalization, *gluing* local rings over an arbitrary ring *A*

SCHEMES

sheaves with basis *Spec(A)*={ *P*: *P* prime ideal in *A*} (adequate topology), fibers={A_P local rings, $P \in$ *Spec(A)*}

algebraic varieties over *k* (char. *p*) ⟷ cohomology groups with coefficients over *F* (char. 0)

relative mathematics: comparisons of schemes

now that the Weil cohomology has to be defined by a completely different approach. Such an approach was recently suggested to me by the *connections between sheaf-theoretic cohomology and cohomology of Galois groups on the one hand, and the classification of unramified coverings of a variety on the other* (as explained quite unsystematically in Serre's tentative Mexico paper[12]),

2.3. The *Éléments de géométrie algébrique (EGA)* (1960-67)

1960-67 **[1960-67] *Éléments de Géométrie Algébrique*** (with Jean Dieudonné), IV volumes (8 parts), Paris: IHES, 1960-1967.

INSTITUT
DES HAUTES ÉTUDES
SCIENTIFIQUES

ÉLÉMENTS DE GÉOMÉTRIE ALGÉBRIQUE

par A. GROTHENDIECK

Rédigés avec la collaboration de J. DIEUDONNÉ

I

LE LANGAGE DES SCHÉMAS

1960

PUBLICATIONS MATHÉMATIQUES, N° 4

5, ROND-POINT BUGEAUD — PARIS (XVI^e)

A titre informatif, nous donnons ci-dessous le plan général prévu pour ce Traité, d'ailleurs sujet à modifications ultérieures, surtout en ce qui concerne les derniers chapitres :

Chapitre Premier. — Le langage des schémas.
— II. — Étude globale élémentaire de quelques classes de morphismes.
— III. — Cohomologie des faisceaux algébriques cohérents. Applications.
— IV. — Étude locale des morphismes.
— V. — Procédés élémentaires de construction de schémas.
— VI. — Technique de descente. Méthode générale de construction des schémas.
— VII. — Schémas de groupes, espaces fibrés principaux.
— VIII. — Étude différentielle des espaces fibrés.
— IX. — Le groupe fondamental.
— X. — Résidus et dualité.
— XI. — Théories d'intersection, classes de Chern, théorème de Riemann-Roch.
— XII. — Schémas abéliens et schémas de Picard.
— XIII. — Cohomologie de Weil.

En principe, tous les chapitres sont considérés comme ouverts, et des paragraphes supplémentaires pourront toujours leur être ajoutés ultérieurement ; de tels paragraphes

(A suivre.)

2.4. The *Séminaire de Géométrie algébrique* (1960-69)

1960-69 **[1960-69] *Séminaire de Géométrie Algébrique du Bois-Marie*** (with diverse co-authors), VII volumes (12 parts), Berlin: Springer, 1970-1973 (original multicopies, IHES, 1960-1969).

SEMINAIRE DE GEOMETRIE ALGEBRIQUE DU BOIS MARIE

1960-61

REVETEMENTS ETALES ET GROUPE FONDAMENTAL

(SGA 1)

un Séminaire dirigé par

A. GROTHENDIECK

SGA 1. Revêtements étales et groupe fondamental, 1960 et 1961.

SGA 2. Cohomologie locale des faisceaux cohérents et théorèmes de Lefschetz locaux et globaux, 1961/62.

SGA 3. Schémas en groupes, 1963 et 1964 (3 volumes), en coll. avec M. DEMAZURE.

SGA 4. Théorie des topos et cohomologie étale des schémas, 1963/64 (3 volumes) (en coll. avec M. ARTIN et J. L. VERDIER).

SGA 5. Cohomologie ℓ-adique et fonctions L , 1964 et 1965 (2 volumes).

SGA 6. Théorie des intersections et théorème de Riemann-Roch, 1966/67 (2 volumes)(en coll. avec P. BERTHELOT et L. ILLUSIE).

SGA 7. Groupes de monodromie locale en géométrie algébrique.

SEMINAIRE DE GEOMETRIE ALGEBRIQUE DU BOIS-MARIE

1963-1964

THEORIE DES TOPOS ET

COHOMOLOGIE ETALE DES SCHEMAS

(SGA 4)

Un séminaire dirigé par

M. ARTIN, A. GROTHENDIECK, J.L. VERDIER

Avec la collaboration de

N. BOURBAKI, P. DELIGNE, B. SAINT-DONAT

TOME 1

THEORIE DES TOPOS

(Exposés I à IV)

EXPOSE IV : "TOPOS", par A. Grothendieck et J.L. Verdier

IHES School (Grothendieck, Artin, Giraud, Verdier) SGA 1962-63

Grothendieck topologies

in categories with enough exactness properties:

abstract notion of *covering* (morphisms stable families) / *sieves* (morphisms ideals)

abstract notion of *topology* (collection *J* of coverings / sieves)

sites (categories *C* with Grothendieck topologies)

abstract notion of **sheaf** : presheaf (functor $C^{op} \to Set$) which *equalizes* appropriate coverings

Grothendieck Topos: topos *Sh(C,J)* of sheaves over a site

$$Sh(C, J) \underset{\text{reflection}}{\overset{\text{inclusion}}{\rightleftarrows}} PreSh(C)$$

Giraud (small): reflexive subcategories come from topologies

Giraud (big): intrinsic characterization of Grothendieck topoi

2.5. The diagrammatic imagination (b): types and archetypes

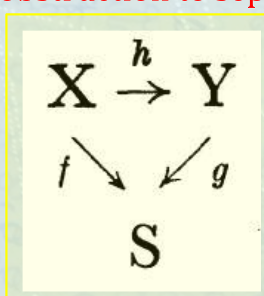
universal extensions of number

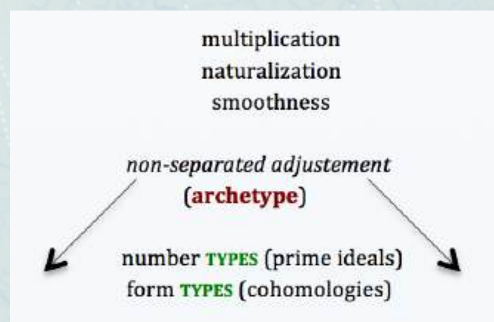
NOT: separated (real) integers — **YES:** glued (ideal) sections

"metaphysical" inversion:	(types)	(archetypes)
methodological inversion:	(statics)	(dynamics)

the archetype (= ***arkhê***; *ark-* Greek root; *arkeô* = move away; *akhô* = found; *arkhên* = project) emerges through the *transformations* of types and the associated *invariants* of transformations ("Serre's schémas", categories of schemes, and Grothendieck's **morphisms variations**)

EGA: obstruction to separation





2.6. The diagrammatic imagination (c): types and archetypes

universal extensions of space

	NOT: points	YES: sections
"metaphysical" inversion:	(types)	(archetypes)
methodological inversion:	(statics)	(dynamics)

the archetype (= ***arkhê***; *ark*- Greek root; *arkeô* = move away; *akhô* = found; *arkhên* = project) emerges through the *transformations* of types and the associated *invariants* of transformations ("Grothendieck's yoga", variable sets, and Grothendieck's **common roots for number/space**)

SGA: obstruction to classical punctuality

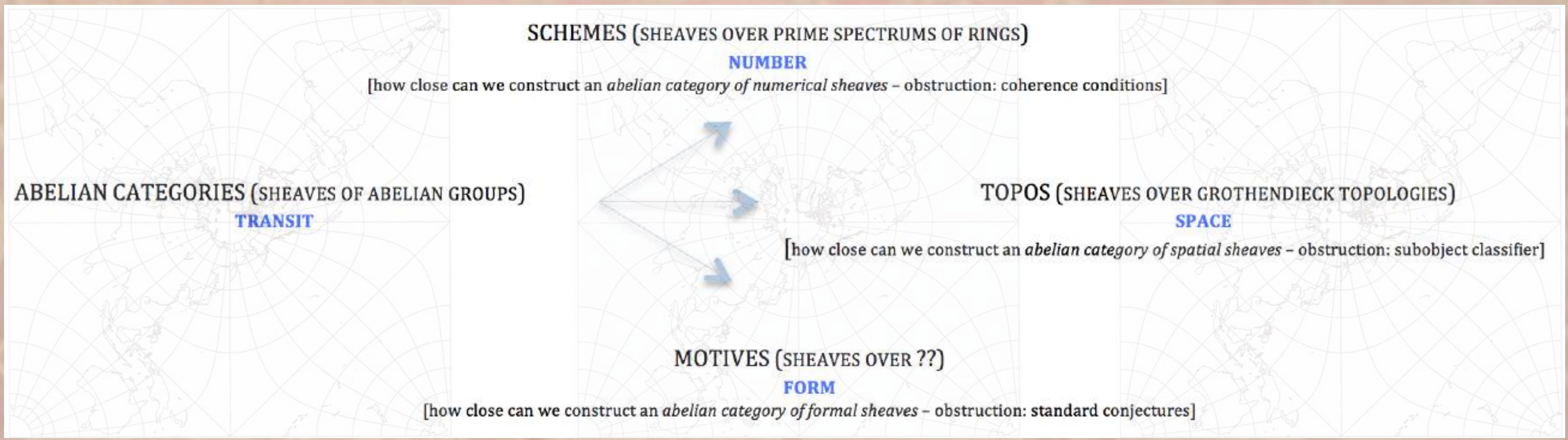
7.4. <u>Topos non vides sans points</u>

multiplication
naturalization
smoothness

non-punctual adjustement
(**archetype**)

space **TYPES** (sites)
form **TYPES** (sheaves over a site)

3.1. A map of main Grothendieck contributions: transit, number, space, form





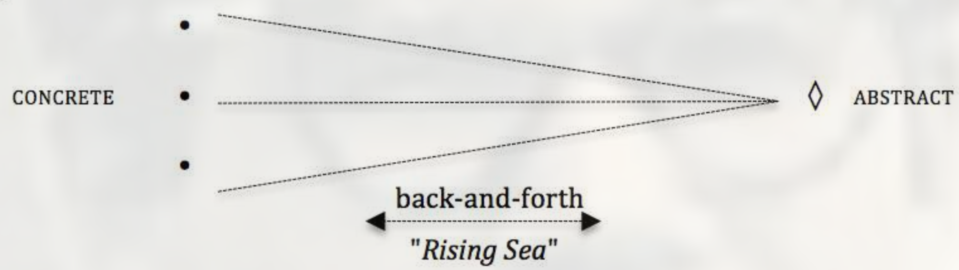
3. 2. Some methodological strategies

FROM THE "THING-IN-ITSELF" TO THE "PROCESS-IN-OTHERNESS" *injections*

particular object	multiplicity of similar objects	associated category
one (basic level)	*Many (intermediate level)*	*One (superior level)*
artificiality		naturality
singularity		smoothness

FROM "TYPES" TO "ARCHETYPES"

projections



TOWARDS NON-SEPARATION

TRANSITS (complexity) : not entirely determined by (separated) neighborhoods : **SHEAVES**

NUMBER (arithmetic) : not entirely determined by (separated) integers : **SCHEMES**

SPACE (geometry) : not entirely determined by (separated) points : **TOPOS**

FORM (topology) : not entirely determined by (separated) groups : **MOTIVES**

3. 3. Back-and-forth along Grothendieck's *third decade* (1981-1991)

THIRD DECADE (C. 1975-1991)

POLARITIES (YIN/YANG, CONCRETION/ABSTRACTION)
AND
CATEGORICAL MEDIATIONS

(A)

NEW STYLE:

PLASTIC WORK DIARIES (YIN), NON ARCHITECTONIC (YANG)

- place to mistakes, amendments, progressive variations
- open spaces: discovery/invention + reflection

(B)

"DIRECT" ACCESS TO RICH ARITHMETIC-GEOMETRIC OBJECTS:

NEW UNDERSTANDING OF NUMBER/SPACE FROM ***CONCRETE DEPTHS***

- moduli spaces of Riemann surfaces
- modular group / absolute Galois group
- Teichmüller tower and complex variables
- topological surfaces
- *dessins d'enfants*

(C)

UNIVERSAL/ARCHETYPICAL ACCESS TO HOMOTOPY:

AXIOMATIC TRANSITS IN CAT FROM ***GENERIC DEPTHS***

- localizations*:* n-groupoids, homotopical n-types, n-stacks
- smoothness and globalization: derivators

(D)

COINCIDENTIA OPPOSITORUM

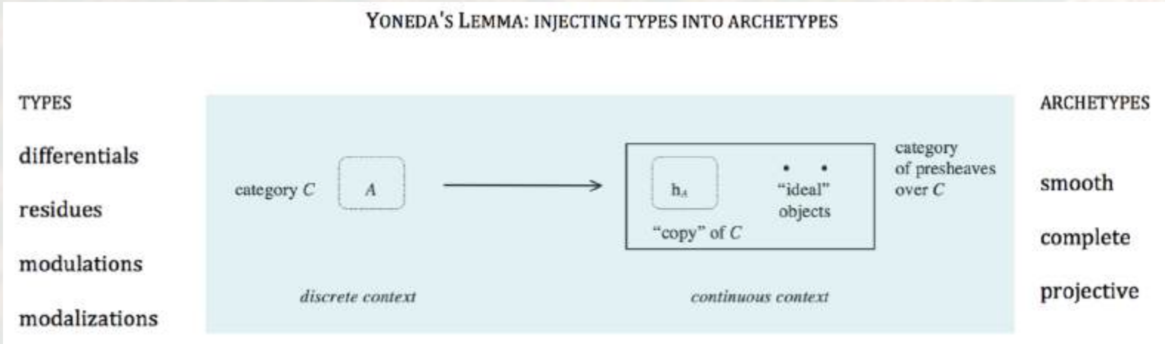
"to see *Cat* with a geometer eye"

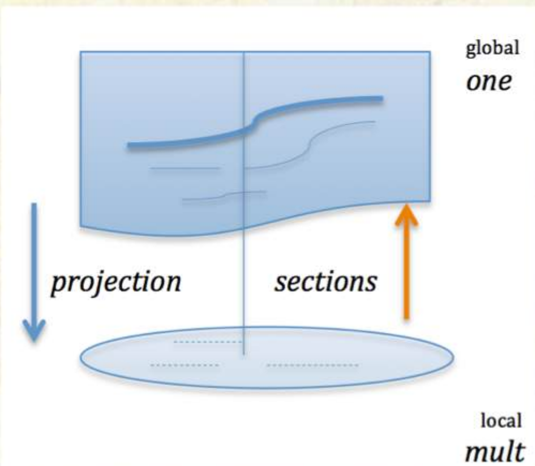
CAT-GEOM

particular (B) ← CAT-GEOM → general (C)

tension / harmonization / smoothness between the polarities

4.1. Grothendieck lenses (a)






Grothendieck – Kansas 1955

Elementary Topos (Lawvere, Tierney 1969-70; Kock, Freyd 1971-72; Johnstone 1977)

program of first-order (elementary) category-theoretic axiomatizations

algebraic theories 1963	*Set* 1963	*Cat* 1966

cartesian closed categories with subobject classifier (Sub representable: $Sub(-) \cong Mor(-,\Omega)$)

+ limits *elementary topos E*

elementary category of **sheaves** (1970)

elementary topology: $j:\Omega \to \Omega$ such that $jT = T$ $jj = j$ $j\wedge = \wedge(j \times j)$

elementary sheaf: characterization through extension of j-dense morphisms

$Sh(E, j)$ inclusion → E ← reflection

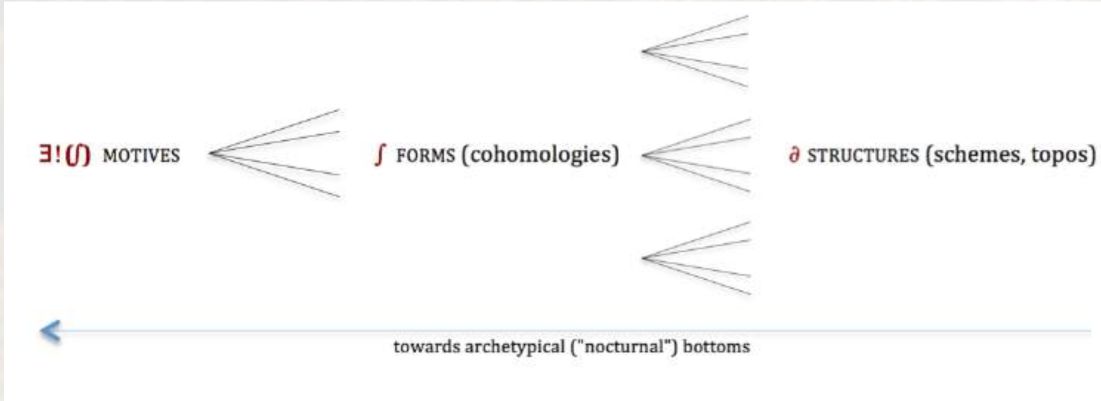
double negation topology $j = \neg\neg$ — **double negation sheaves** $E = Sh(Set^{C^{op}}, \neg\neg)$ (Fourman 1977-80: equivalent to Boolean valued models)

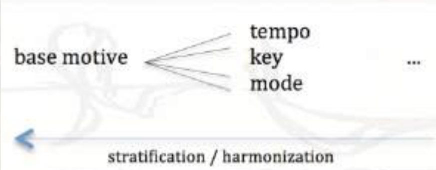
Freyd (1980: independence axiom choice)

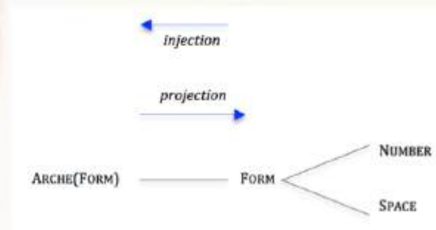
C ← *non-zero finite ordinals / left inverses to inclusions*; *non-zero finite ordinals / appropriate tuples*; *monoid of 1-s words and appropriate equations* | *E forces* $\neg AC$


Grothendieck - Bombay 1968

4.2. Grothendieck lenses (b)







Grothendieck – Montréal 1970

4.3. Grothendieck lenses (c)

"The *new geometry* is before all
a **synthesis** between the arithmetical world and the world of continuous magnitudes –
those worlds, **separated before**, now constitute only one"

[Grothendieck, 1985]

GALOIS
DISCR
algebraic varieties
number

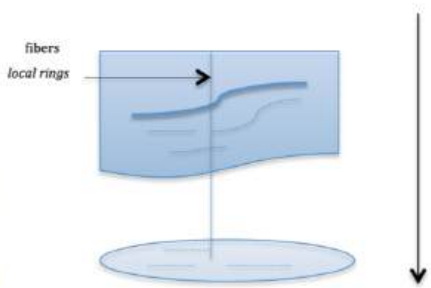
GROTHENDIECK
SCHEMES
base changes
functoriality

RIEMANN
CONT
differential varieties
magnitude

SCHEMES
(Cartan, Chevalley, Serre)
GROTHENDIECK

(1) AFFINE

consider general commutative rings
observe NOT: points – YES: prime ideals



spectrum
prime ideals
Zariski topology

(2) GENERAL

GLUE (LOCAL) AFFINE SCHEMES

CONCEPTUAL REVOLUTIONS

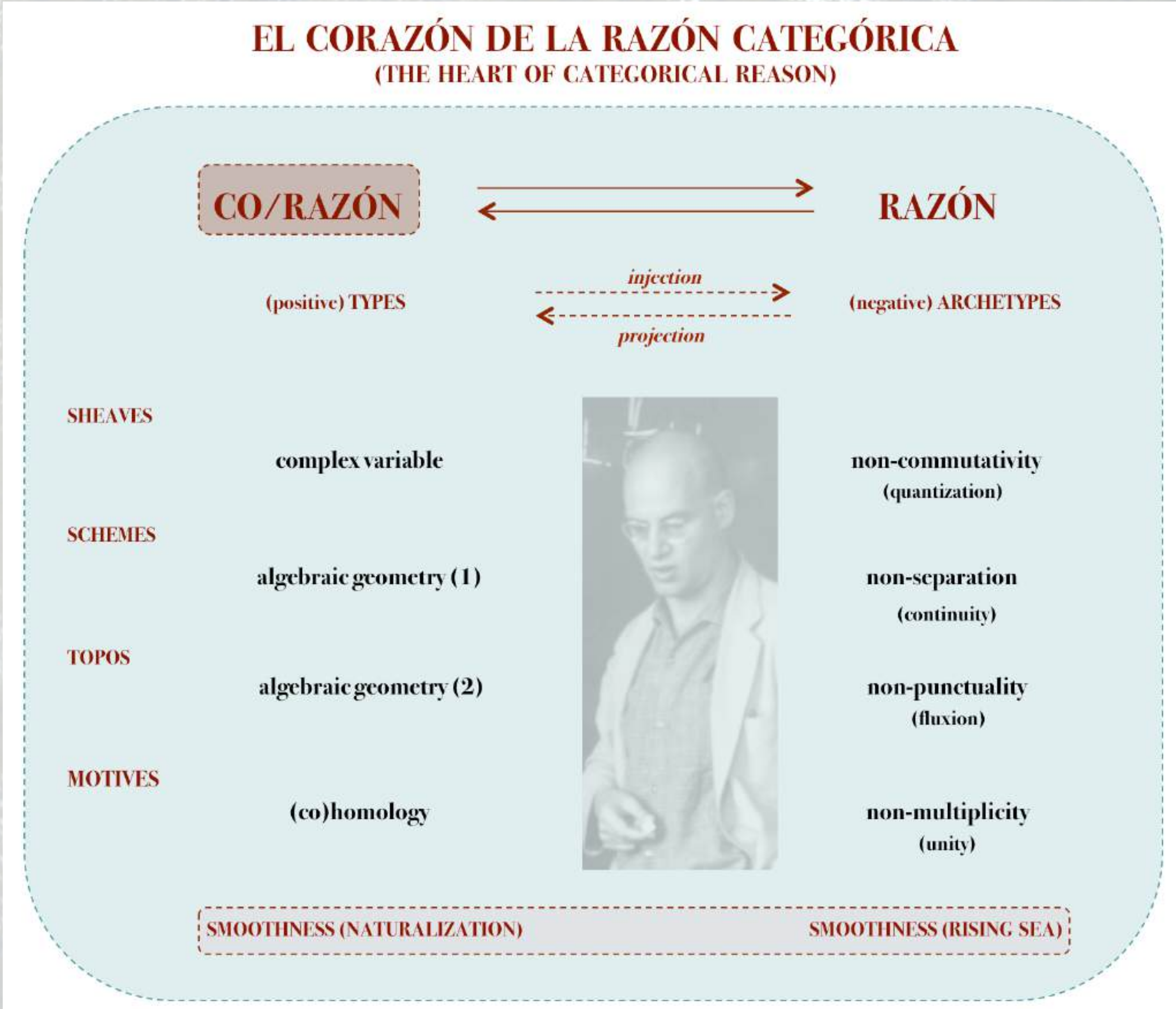
disappearance of classical, separated points
emergence of non-classical, continuous "points"

objects understood through
their *differential residues* and their *integral aura*:
substitution of the object X (and its classical points $1 \rightarrow X$)
by its representable functor h_X (and its non-classical points $T \rightarrow X$)

double gluing
(local/affine and global/general)

Grothendieck – Paris, 1963

4.4. Non-Separated "Razón / Co-razón"



4.4. Non-Separated “Razón / Co-razón”

“Grothendieckean Lemmas”

UNITY is revealed through **MULTIPLICITY**

ABSTRACTIONS dissolve **OBSTRUCTIONS**

DEEPNESS enhances **SOFTNESS**