Letter to J. Lipman, 12.6.1969

par

Alexander Grothendieck

Transcription by

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Massy 12.6.1969

Dear Lipman,

Thanks for your letter. The answer to your question whether $\hat{\hat{A}}[[T]]$ factorial implies A has a rational singularity is affirmative, at least in the equal characteristic case. This comes from the construction of the local Picard scheme scheme G over the residue field in this case (cf. SGA 2 XIII 5), as it is easily checked that A has a rational singularity if and only if G is of dimension zero. In the contrary case, as the neutral component G° is smooth of dim > 0, there would exist a non constant formal arc passing through the origin of G°, and it is easily seen that this arc defines a non trivial divisor class of $A[[T]]$. This argument will work in the unequal characteristics case too, provided we can extend to this case the construction of the local Picard scheme (and its universal property). I hope this could be done, at least for a perfect residue field, but never checked this point. (I proposed this to Lichtenbaum seven years ago, but I am afraid he never looked into this!)

As for the question you mention concerning the $\mathrm{H}^1(Z,\underline{O}_Z)$ of the Zariski-Riemann space Z of A, I confess I have not much feeling for that animal, but I guess this is equivalent with the direct limit of the $\mathrm{H}^1(X,\underline{O}_X)$ for all models birational and proper over $\mathrm{Spec}\,A$ (which, in case we have resolution, would be also the H^1 of any regular such model). The idea that this direct system might be essentially constant, and that this may be used to prove resolution, had been mentioned to me also, five or six years ago, by Artin, but I believe he could not push it through. An analogous problem, whose solution would be needed in order to construct local Picard schemes for noetherian local (excellent ?) rings of higher dimension, is the following: does there exists a birational proper model X of $\mathrm{Spec}(A) = S$, inducing an isomorphism $X|(S-s) \simeq (S-s)$ ($s =$ closed point), such that every invertible sheaf on $S-s = X - X_0$ extend to an invertible sheaf on X? If s is an isolated singularity, this would follow of course from resolution. In case s is not isolated, it seems the answer is not known even for complete local rings of char. 0.

In any case, I asked Hironaka who does not know.

Sincerely yours,

A. Grothendieck

Comments: 1). Pic of a curve is reduced ($H^2 = 0$) and is smooth.