Letter to J. Murre, 1959 (?)

par

Alexander Grothendieck

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Dear Murre,

Thank you very much for your notes on the tame fundamental group, which I at least finished reading. I see you wrote them with much care, and I am all the more sorry that my own fault, there is a number of misstatements which, I am afraid, will force you to do a serious recasting of the whole exposition. My notes definitely where too sketchy, and my oral explanations, I am afraid, partly wrong, which induced you into error a few times. Here the most serious drawbacks.

1.16 is false already when H is the unit group and when there is a single a, say $a = b^n$, $Y' = YT/(T^n - a)$. Then $Y = S$, and a morphism of Y into Y' compatible with $H = e\ H'$ is just a section of Y', which exists indeed; however $H \longrightarrow H'$ is not surjective. 3.6. is equally false, as you see by the previous example, using the given section to define an H'-morphism $H' \longrightarrow Y'$ which is not an isomorphism. As a consequence, the proof in your notes of 3.7. breaks down (as it uses 1.16) and so does the proof of 3.8. (I did not try to check 3.8. by some different proof).

I am afraid 6.4. is false as stated, and that the statement is correct only if the D_i are regular. Indeed, the end of the proof seemed to me very dubious; be careful that the inertia groups are determined only up to interior automorphism! There is however a (tautological) generalization of the theorem for regular D_i, corresponding to the data of a single divisor D with normal crossings, and a variant of the notion of tame ramification for such a divisor, by demanding that the coverings should be tamely ramified locally for the étale topology for the family of local irreducible components of D; it is this notion of tameness which should seem more adapted to the situation of par. 9.

The proof of 7.1. is not correct, when you contend on line -9: there remains to be proven the following... Already when $D = 0$, the proof here would have to introduce connected étale coverings which are *not Galois!*

This very strongly suggests that a notion of tame ramification should be introduced also for non Galois coverings. The same remark applies t the proof of 10.1. Maybe you could get along some way in 7.1. using the normality assumptions, but I am convinced that these assumptions are anyhow artificial, as well as the assumption that the D_i should be reduced somewhere. You do not seem to make any use of theses facts, really.

Also, one feels that 7.5. should be generalized to the case of a tamely ramified covering, and that it should come out trivially once the generalities have been dealt with properly.

I hope that the theory will come out more clearly and correctly by devoting some care to generalities on ramification data (not necessarily of Kummer type). I will try to write something up within the next days. Please excuse me for the trouble I caused you by not learning my lesson well enough before I put you to work!

It will be very nice indeed to have an appendix on Lefschetz theorem for the fundamental group, and it should not be hard to write it. However, if would be safer to wait till the general theory of tame ramification is written up!

Sincerely yours