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Dear Lipman,

I got a copy of your nice work on rational singularities. Just one comment to the "main unanswered question" on p. (iv) of your manuscript: the answer is quite evidently: no, essentially for the reason you indicate yourself. The simplest example would be to start with an elliptic curve E over a field k, such that E(k) = 0, a torsor (= princ. hom. space) C under E of order $n \geq 3$, and the projective cone of the natural projective imbedding of E of order n (or the smallest multiple

$n'>0$ s. th. $\mathrm{Pic}^{n'}(C) \neq \emptyset$, if $\mathrm{Br}(k)$ is not zero); take the completion of the local ring at the origin of that cone.

This example is also an example where A is factorial, but $A[[t]]$ is not: the main reason is that the local Picard scheme P (cf SGA 2 XIII p. 19) is of dim >0, although $P(k)=0$.

Sincerely yours

A Grothendieck

Comments (Points to be verified)

1) order of a torsor divides index (= g.c.d. of degrees of divisors) with equality if $\mathrm{Br}(k)=0$. (e.g. Columbia notes)

2) torsor (is non-sing.) has genus 1, so any divisor of degree ≥ 3 defines a projective embedding, (even arithmetically normal!)

3) for any non-singular genus 1 ~~elliptic~~ "normal" (in classical sense) curve C, the projecting cone is normal, and ~~local~~ $\mathrm{Pic}(\hat{U}) \cong \mathrm{Pic}^0(C)$ (U = punctured nbd.) cf. Hironaka notes. (not quite: cyclic cokernel of order = degree/index)

4) for any torsor, E itself is the Jacobian (cf. Cassels survey)

Examples for E: (i) Salmon, Rend. Lincei, May 1966
(ii) Cassels ...
(iii) Danilov